A Time-referenced 4D Calibration System for Kinematic Optical Measuring Systems

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Abstract
By using kinematic optical measuring systems in spatiotemporal positioning necessarily all involved sensors of the measuring systems have to be synchronized. Otherwise existing dead time and latency in a measuring system will lead to deviations in the space-time position. A time-referenced 4D calibration system is presented for kinematic optical measuring systems, which is qualified for tracking optical measuring systems of any kind. The base of this calibration system is built up by a tiltable rotating arm driven by a rotary direct drive. The rotating arm is supplemented by a further rotary direct drive mounted on a movable tripod. The developed modeling for determinability of a space-time position is based on the theory of quaternions. The fundamental idea of modeling is equivalent to the fact that every measurand of the test item, which is measured at a particular time, could be assigned to an explicit position of the rotating arm.

Keywords
Kinematic measurements, calibration system, time-referenced, rotating arm

1 INTRODUCTION

Kinematic optical measuring systems such as lasertracker, robotic-tacheometer or iGPS are employed for the space-time position determination of moving object points. These kind of measuring systems are multi-sensor systems and for a spatiotemporal positioning all involved sensors have to be synchronized. It is a fact that existing dead time and latency in a measuring system will lead to deviations in space-time position.

The development of a 4D calibration system is based on a discrete spatiotemporal position determination. By the calibration system the nominal trajectory is representing by a rotating arm and together with a time referencing every position is known in space and time. With an adequate modeling it is possible to determine the relative time for a measuring result of every mesurand of a test item. In this way the relative time gives information about the measuring point of time. It is also possible to determine dead time or latency from these times. In this paper the basics of the calibration system will be represented as well as a part of the modeling. More details are presented in Depenthal (2008).

2 4D CALIBRATION SYSTEM

2.1 Technical Realization

The calibration system is designed by a rotating arm with an arm length of 2m. The arm is rotating in a horizontal or vertical plane and also in planes between both. At the end of the arm a prism or sensor can be fixed and a balance weight on the opposite end. The performance of the prime mover of the rotating arm consists of a rotary direct drive with an integrated rotary encoder. The encoder has a resolution of 0.36" and the grating disk has a reference point, the so-called homepoint, for a defined orientation. After a calibration of the direct drive a measurement uncertainty of $U_{k=2}$ = ± 4.0" is achieved (Depenthal, 2006). In addition a function is generated to correct for the bending of the spatial position of the rotating arm. The direct drive can produce velocities up to 10m/s at the arm's end.

A lasertracker LTD500 (Leica) was used to verify the static accuracy of the rotating arm. In relation to the rotating plane standard deviations are reached as follows: out of plane $\sigma = 17\mu m$, radial $\sigma = 10\mu m$, tangential $\sigma = 9\mu m$. Kinematic measurements until 6m/s exhibit a stable behavior of the rotating arm (Depenthal and Barth, 2007).
The length of the rotating arm restricts the angular range of polar measurement systems. To enlarge the horizontal angle, a larger rotation has to be simulated for the measurement system. This can be reached, if the measurement system is mounted on a rotary direct drive. In this way the measuring system performs the same rotation as the direct drive. This second direct drive is mounted on a very stable and heavy tripod. The direct drive has a resolution of 0.22" and after a calibration a measurement uncertainty of $U_{k=2} = \pm 2.3"$ is achieved.

2.2 Time Referencing

Ideally, a kinematic measurement process has to assign an accurate spatiotemporal position to a moving object. In fact there is a difference between the measured spatiotemporal position and the theoretical position. The dimension depends on the measurement system. A time-referenced calibration system has to detect these differences and to enumerate the dimension.
The meaning of time referencing is that specific procedures have to be kept at the same point of time on a given time scale. For time referencing only real-time systems can be used. A system is said to be real-time if the total correctness of the result of a real-time data processing depends not only upon its logical correctness, but also upon time in which it is performed (Wörn and Bringschulte, 2005). A real-time system also has to be guarantied a temporal deterministical behavior (Mächtel, 2000).
There are two different procedures for time referencing between a calibration system and the test item: external trigger and serial interface. An external trigger is used, if the measuring system has a trigger input interface. The trigger-signal is realized with a function generator, normally using the rising or trailing edge of a rectangular signal as trigger. The quality of the time referencing is only dependent on the edge's quality. The clock rate of the trigger signal must be choosen in that way, that all procedures of the calibration- and measurement system can be closed within one clock rate.
The other method of communication is a serial interface. The communication between the two participants – measuring system and calibration system – is made up of requests and replies in terms of the data item. Thereby the trailing edge of each start bit of the data item – request and reply – will be captured. Assuming that the data transfer rate would be 19200 baud, the period between two trailing edges constitutes 103µs so that the trailing edge of the start bit must be captured within this period. The calibration system assigns a position – time and location – to the respective start bit and the result is a spatiotemporal position for every request and reply of a measurement system.

2.3 Control System

The main item of the control system is the real-time multi-axis servo motion controller PMAC (Programmable Multi-Axes Controller). Its power and speed allows handling of complex motion sequences and it is used for the position and velocity control of the direct drives. The position-capture function latches the current encoder position at the time of an external event into a special register. The actual latching is executed in hardware, without the need for software intervention. This means that the only delays in a position capture are the hardware gate delays (less than 100ns) thereby providing a very accurate capture function.
For the realization of the time referencing with a serial interface the trailing edge of a start bit must be detected within 103µs. Furthermore, a trigger signal must be send at the same time to PMAC, in order that every start bit gets an encoder value. This task must be solved in real time, therefore a FPGA-modul (Field Programmable Gate Array) is used. The synchronization of the I/O signals and the acquisition of the temporal processes are carried out with a resolution of 25ns. In order that an edge can be captured by the FPGA a level converter must be switched between. The time referencing is defined by 1µs (worste case) with a measurement uncertainty $U_{k=2} = 0.1\mu s$. Figure 1 shows the single components of the calibration system with a robot-tacheometer as exemplary test item.

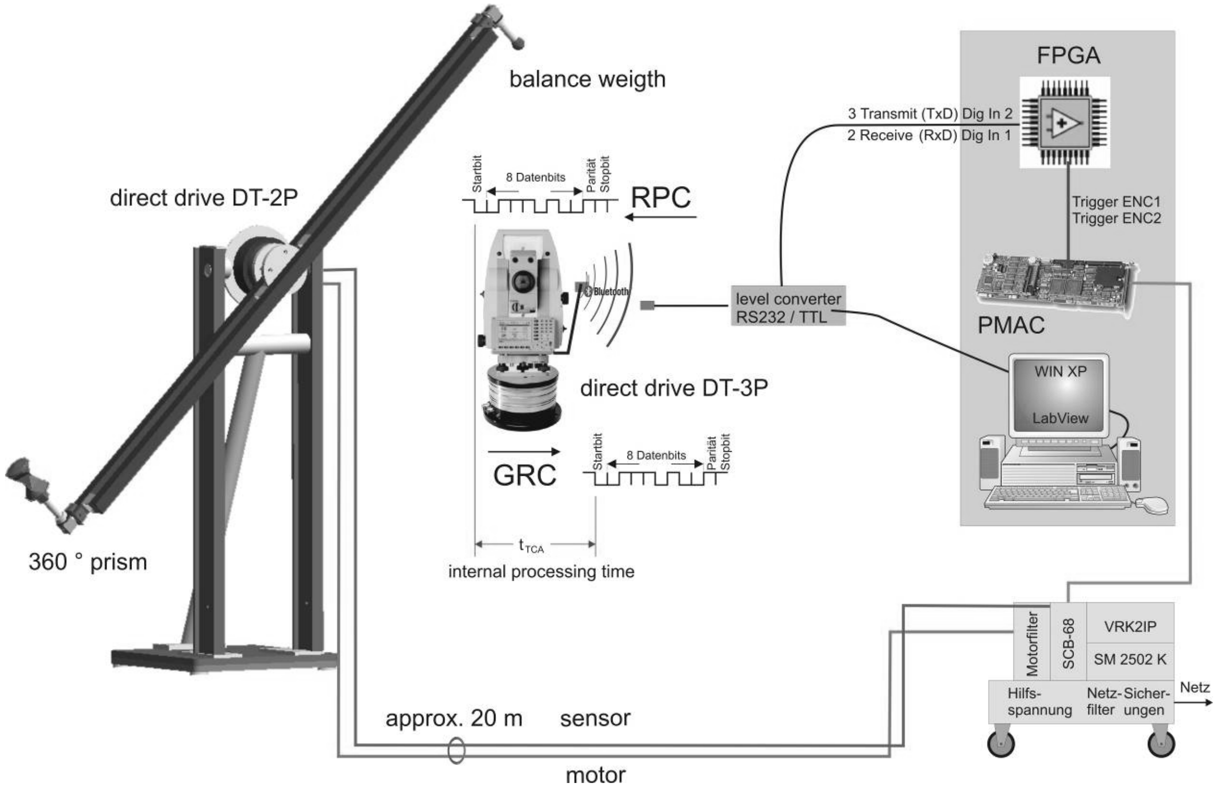

Figure 1: single components of the calibration system with a robot-tacheometer as exemplary test item

3 KINEMATIC MODELING

3.1 Principle

The modeling approach is based on the fact that every measurand of a measuring system is measured at a specific point of time. The time-referenced rotating arm can be assign this specific point of time to an exactly defined rotation angle in respect of the reference position. By known angular velocity and rotation angle it is possible to draw a conclusion for the searched time of the measurand.

Kinematic measurements are characterized by existence of no repeated measurements. Therefore every single point of measurement must be examined and the model must bear as unique unknown the rotation angle $\varphi(t)$ which is assigned to the measurand. The modeling is based on the theory of quaternions, which are given, for example, in Kuipers (1999). A quaternion may be regarded as 4-tupel of real numbers, that is an element of R^4. A quaternion is defined as a sum of a scalar part and a vector part, which is an ordinary vector in R^3. The multiplication of quaternions is non-commutative. The advantage of using quaternions is the efficient concatenation of multiple rotations.

First of all, the radius of the rotating arm will be determined with a static reference measurement due to the higher precision of the measurements in the static mode of the measurement systems. The test item and the calibration system exhibit their own co-ordinate system and a co-ordinate transformation will be calculated using quaternions (Horn, 1987).

The aim of modeling is to transfer the co-ordinates of the test item in a function of the angle $\varphi(t)$. Then the latency of each measurand can be derived. For a clear reference the homepoint of the direct drives are used.

3.2 Spatial Movement with One Rotation

In this paper the focus is the description of the modeling for polar measuring systems. The modeling for measuring systems with solely angular or distance measuring are described in Depenthal (2008).

Starting at point $\mathbf{p}_{\mathbf{D,1}}=(r,0,0)^T$ with the radius r of the rotating arm, every new circle position results from the rotation angle $\varphi(t)$ in the circle plane. The rotation axis is equivalent to the z-axis of the rotating arm system and the quaternion q is set up in the following form

$$q = \left(\cos\left(\frac{\varphi(t)}{2}\right), \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \sin\left(\frac{\varphi(t)}{2}\right) \right) . \tag{1}$$

After rotating the starting point with the angle φ in positive direction the new position $\mathbf{p}_{\mathbf{D,2}}$ is build through a quaternion multiplication as

$$p_{D,2} = q\, p_{D,1}\, q^* \tag{2}$$

with $p_{D,1}$ as a pure quaternion.
By the rotation-quaternion q_R and the translation-quaternion q_{tr} which are determined by the co-ordinate transformation, the position $p_{D,2}$ will be transformed from the rotating arm system to the co-ordinate system of the test item with the following form

$$p_{P,2} = q_R\, p_{D,2}\, q_R^* + p_{tr} = q_R\, q\; p_{D,1}\, q^* q_R^* + p_{tr} = q_R\, q\; p_{D,1} (q_R\, q)^* + p_{tr} \quad . \tag{3}$$

The new position (3) can be directly assigned as a vector $\mathbf{p}_{\mathbf{P,2}}$ in R^3

$$\mathbf{p}_{\mathbf{P,2}} = \begin{pmatrix} (2(q_{R,0}q_0 - q_{R,z}q_z)^2 - 1 + (q_{R,x}q_0 + q_{R,y}q_z)^2)r + p_{tr,x} \\ (2(q_{R,x}q_0 + q_{R,y}q_z)(q_{R,y}q_0 - q_{R,x}q_z) + 2(q_{R,0}q_0 - q_{R,z}q_z)(q_{R,z}q_0 + q_{R,0}q_z))r + p_{tr,y} \\ (2(q_{R,x}q_0 + q_{R,y}q_z)(q_{R,z}q_0 + q_{R,0}q_z) - 2(q_{R,0}q_0 - q_{R,z}q_z)(q_{R,y}q_0 - q_{R,x}q_z))r + p_{tr,z} \end{pmatrix} \tag{4}$$

$$q_0 = \cos\left(\frac{\varphi(t)}{2}\right) \qquad q_x = 0 \qquad q_y = 0 \quad \text{und} \quad q_z = \sin\left(\frac{\varphi(t)}{2}\right) \quad . \tag{5}$$

The unique unknown in (4) is the quaternion q with the separate elements (5), which will be inserted in (4). In this way every measured point can be described by the rotating angle $\varphi(t)$.
The next step is to assign the result of measurement to the respective position of the rotating arm. Every polar mesurand can be calculated from vector (4). The non-linear equations cannot be solved analytical, but must be solved numerically by Newton's method. The iterative solution follows from the definition of recursion of Newton iteration

$$\varphi(t_i)_{k+1} = \varphi(t_i)_k - \frac{f(\varphi(t_i)_k)}{\dot{f}(\varphi(t_i)_k)}, \qquad \dot{f}(\varphi(t_i)_k) \neq 0, \quad k = 0,1,2,..... \tag{6}$$

The angle $\varphi(t_i)$ describe the different measurement points of time. The time-referenced measurement has enabled to define an initial value for the angle $\varphi(t_i)$. This value will be close to the unknown value and a solution will be found after few iteration steps.
As example, for a distance measurement s at the unknown point of time t_1 the function from algorithm (6) has the following form

$$f(\varphi(t_1)) = p_{P,2,X}^2 + p_{P,2,Y}^2 + p_{P,2,Z}^2 - s(t_1)^2 = 0 \quad . \tag{7}$$

The result of (6) with the function (7) is the angle $\varphi(t_l)$ and with the known angular velocity of the rotating arm the time t_l – the point of time of distance measurement – can be determined.
The same procedure can be used with the other polar measurands.

3.3 Spatial Movement with Two Rotation

In 2.1 it is described that the test item is mounted on a rotary direct drive to enlarge the horizontal angle of a polar measurement system. If the test item – in this case a robot-tacheometer – has locked a static prism and the direct drive starts rotating clockwise (Fig. 2, (1)), then the test item must countersteer to keep the prism (Fig 2, (2)). The acquired data deliver a circle with the test item in the center and the radius equal to the measured distance. A movement of the prism causes a new distance and therefore varies the circle.
The co-ordinate system of the test item is used for the rotation of the direct drive and the Z-axis of the test item is equal to the rotation axis for the quaternion q_1 with the rotation angle $\gamma(t)$ at the point of time t

$$q_1 = \left(\cos\left(\frac{\gamma(t)}{2}\right), \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \sin\left(\frac{\gamma(t)}{2}\right) \right) . \tag{8}$$

The second rotation (Fig. 2, (3)) results from the rotating arm and the quaternion q_2 has the same form as (1)

$$q_2 = \left(\cos\left(\frac{\varphi(t)}{2}\right), \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \sin\left(\frac{\varphi(t)}{2}\right) \right). \tag{9}$$

Now the test item tracks the prism of the rotating arm and through the rotation of the direct drive under the test item, there is an additional change of the horizontal angle. The resulting trajectory is determined in dependence of the velocity of both direct drives and by the spatial position of the rotating arm. The spatiotemporal definition of both rotations delivers a 4D reference position. The starting point is again the vector $\mathbf{p}_{\mathbf{D},\mathbf{1}}=(r,0,0)^T$ with r as radius of the rotating arm. The new position $p_{D,2}$ is reached by the following tripleproduct

$$p_{D,2} = q_2 p_{D,1} q_2^* . \tag{10}$$

A static reference measurement determined the quaternion q_R and q_{tr} for rotation and translation between the both co-ordinate systems. By this way the point $p_{D,2}$ can be transformed in the test item system as point $p_{P,2}$

$$p_{P,2} = q_R p_{D,2} q_R^* + q_{tr} = q_R (q_2 p_{D,1} q_2^*) q_R^* + q_{tr} . \tag{11}$$

In next step this point will be rotated by the angel $\gamma(t)$ with the quaternion q_1 as following

$$p_{P,3} = q_1 p_{P,2} q_1^* = (q_1 q_R q_2) p_{D,1} (q_1 q_R q_2)^* + q_1 q_{tr} q_1^* = q_3 p_{D,1} q_3^* + q_1 q_{tr} q_1^* . \tag{12}$$

Similar to (4) the quaternion (12) can be assigned as 3D-vector. The angles will be replaced by

$$\varphi(t) = \omega_D \, t \tag{13}$$
$$\gamma(t) = \omega_P \, t \tag{14}$$

and respectively, with the known angle velocities the time t is the unique unknown. The searched time for every measurand will be determined again by Newton's method.

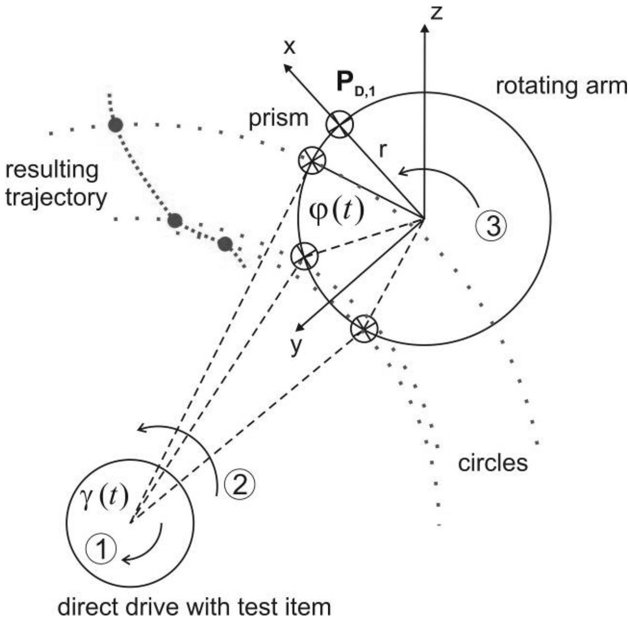

Figure 2: principle of two rotations

The maximum dimension of a trajectory, which may be reached with two rotations, is bound to the maximum distance between test item and prism and to the diameter of the rotating arm. A simulation can assess the quality of the trajectory. Figure 3 shows the trajectory, which results from an angular velocity of 30°/s for the rotating arm and 50°/s for the test item. The trajectory will repeat itself after three circulations of the rotating arm. Another example shows figure 4 with a horizontal rotating arm and an angular velocity of 32°/s for the rotating arm and 10°/s for the test item. The figure shows seven circulations of the rotating arm and the trajectory offers piecewise-linear approximation.

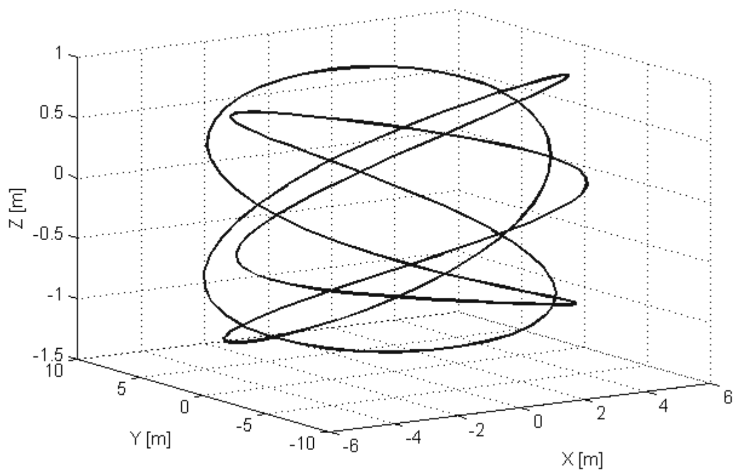

Figure 3: vertical rotating arm, 3 circulations, angular velocities ω_D = 30°/s and ω_P = 50°/s

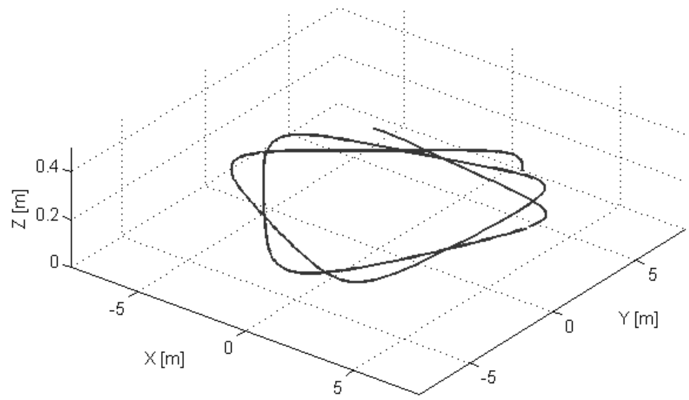

Figure 4: horizontal rotating arm, 7 circulations, angular velocities ω_D = 32°/s and ω_P = 10°/s

4 TEST MEASURING

An exemplary calibration sequence and a first result for Leica robot-tacheometer TCRA1201 (short: TCA) will be represented in the following. The time is referenced by the serial interface (RS232). For the communication between calibration system and TCA Leica's GeoCOM ASCII interface is used, which is a point-to-point connection. In this way a request (RPC) will be sent over the serial interface to the listening TCA and a reply (GRC) will be received. During the time period between RPC and GRC the TCA must be execute the procedure call, in this case: execute a measurement. For the time referencing the FPGA captures the first trailing edge of start bit of RPC and at the same time the PMAC gets a trigger from FPGA. Therefore, encoder value with the corresponding time is known (Fig. 5). The same procedure is carried out with the reply GRC. For a complete measurement the measurands – distance (s), horizontal (HZ) and vertical angle (V) – must be measured between RPC and GRC. Otherwise, the measurands cannot be regarded as actual values for this measurement.
The allocation of the measured value for the position at the rotating arm is carried out according to the modeling for polar measuring systems. The angle φ_{RPC} is used as a start value for Newton iteration. By the known angular velocity the unknown points of time (t_{HZ}, t_V, t_s) can be estimated using the angles φ_{HZ}, φ_V and φ_s respectively (Fig. 5).

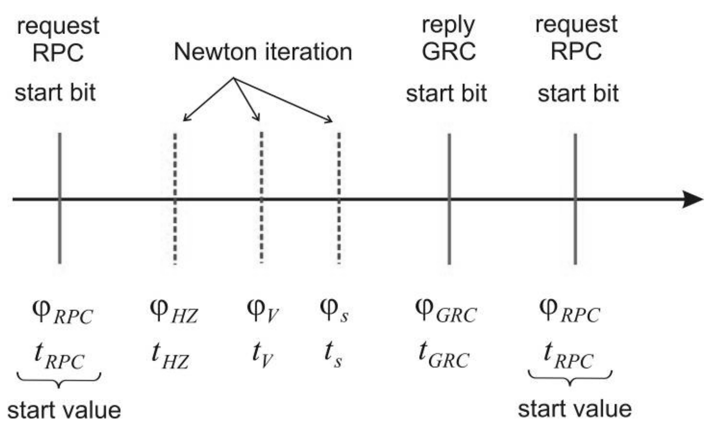

Figure 5: time referencing serial interface by the example of a Leica robot-tacheometer

Before the kinematic measurement is started, the transformation parameters must be determined by static measurements. Figure 6 sketches an example with the rotating arm arranged in a horizontal position and the distance between the homepoint and TCA being nearly 6m (Fig. 6). Starting at homepoint every 30° a point is measured. Figure 7 shows the residuals after the co-ordination transformation. The residuals represent the sum of the accuracy of the measurement system within this distance and the accuracy of the transformation.

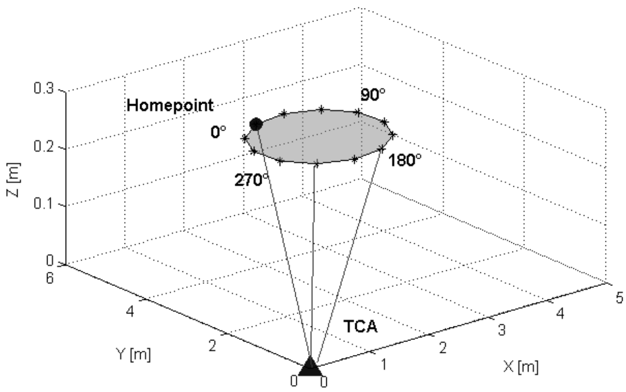

Figure 6: measuring constellation

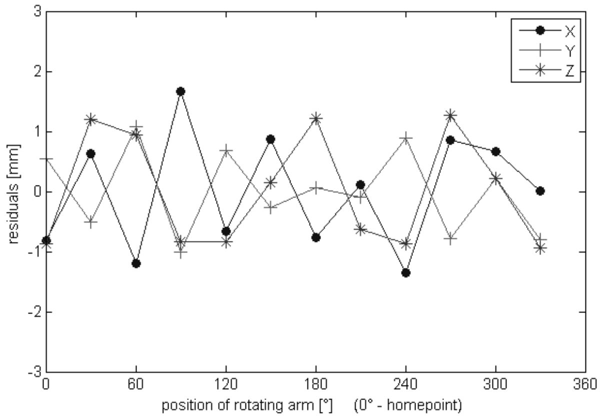

Figure 7: residuals of the static measurement

The kinematic measuring is executed by a rotating velocity of 40°/s. Figure 8 shows the result for one complete rotation of the rotating arm. The homepoint is equal to 0° position. The RPC time will be set to zero for every single measurement. Therefore, the calculated times of every measured value and GRC time will be related to RPC time. The GRC times vary between 90ms and 170ms (Fig. 8).
For a calibration the use of the terms dead time and latency are to consider. It is more useful to use the point of time of RPC as reference and than all calculated delay times apply to this RPC time. A 4D position can then be related to this RPC point. In this context the calculated times are only defined as delay times.
Normally, all calculated times should be located between RPC and GRC. Figure 8 clearly shows that only the values for the horizontal angle lies within this area. The delay time varies between 34ms and 113ms. The distance measurement behaves different. Only few values are located between the area PRC – GRC. Most values are assigned to a position which lies before the actual request of measurement. That means, that the distance cannot be measured directly, but the value correspond rather to an achieved or averaged value.
The represented measurement uncertainties for every calculated time value (Fig. 8, scale x4) indicate the dependence on the position of the test item in relation to the rotating arm position. The measurement uncertainty also depends on the velocity of the rotating arm. The measurement uncertainty enlarges with a slow rotation.
Within this constellation an allocation for the vertical angle is not suggestive because the angel variation is too small and therefore the measurement uncertainty too large.
These results reveal that the time-referenced calibration system can detect a delay time for every measurement value of a measurand in relation to a definite request time. In this process a delay time also can detect before the actual request of measurement started.

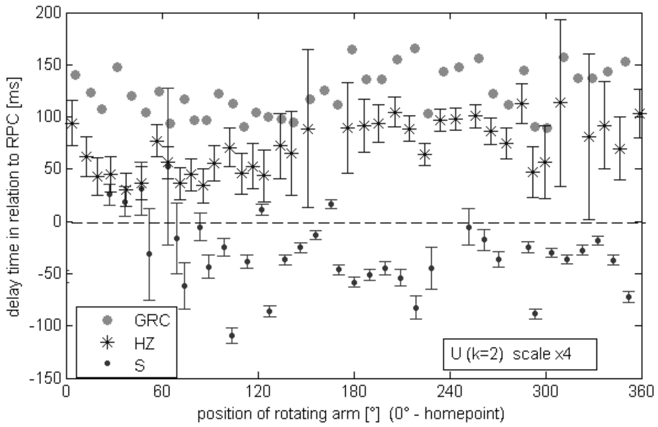

Figure 8: kinematic measuring by a rotating velocity of 40°/s (horizontal rotating arm), GRC time, delay time for horizontal angle and distance. All times in relation to RPC. Measurement uncertainty scale x4

5 CONCLUSIONS

In this paper the development of a time-referenced 4D calibration system and corresponding modeling for polar measuring systems was shown. The 4D calibration system can be used for different kinematic optical measuring systems. The rotating arm bases on a rotary direct drive and a real-time control system for position and time. The time referencing with trigger or with serial interface can assign an encoder value and a relative point of time to a corresponding event. The rotating arm is supplemented by another rotary direct drive, which is mounted on a tripod and which already enable a support for the test item.

First measurings have shown the successful conception of the calibration system in combination with the modeling based on the theory of quaternions. In this way, every measurand can be assigned by a time-referenced position and therefore a delay time in relation to the measuring request can be calculated.

In the near future, it is planned to analyze these results and to develop an adequate calibration function for the existing delay times. Also, it is planned to use the calibration system with other test items than the polar measuring systems.

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